elliptic wing. Figures 1a and 1b show the spanwise distributions of the sectional induced drags for the cases of Table 1. The dashed lines in Fig. 1a are analytical results based on Hauptman and Miloh's method,[9] which produces the closed solution of the linearized lifting surface problem for the thin elliptic wings in steady incompressible potential flow. As can be seen from these figures, both of two numerical methods not only agree well with each other in the far- and near-field cases, but almost agree with the theoretical results in Ref. 4. It is noticed that neither of the numerical far-field induced drag distributions for the circular wing follow the analytical result based on Hauptman and Miloh's method.[9] Actually, when applying their method to the procedure in Ref. 4, the total induced drag coefficients of the circular wing with the angle of attack being 1 rad are

$$C^N_{\mathrm{Di}} = C_L^2/4 = 0.80169\ldots$$

$$C^F_{\mathrm{Di}} = \frac{C_L^2}{16}\sum_{j,k=0}^{\infty}\frac{1}{(2j+1)(2k+1)}\left(\frac{2}{2j-2k+1} - \frac{1}{j+k+2} - \frac{2}{2j-2k-1} + \frac{1}{j+k+1}\right) = 0.80313\ldots$$

where C_L should be taken as the value predicted by their method, $32/(8+\pi^2)$. The two values coincide with each other only to the second decimal place, whereas the corresponding values in Table 1a do so to three places. This result indicates that their method may be approximate because it does not satisfy the fact that the near- and the far-field induced drags should agree in the thin wing theory.

Conclusions

This Note has investigated the accuracy in the induced drag calculation using the discrete numerical lifting surface methods, QVLM and BISM. The formula presented for estimating the leading-edge thrust distribution by the latter numerical method has proven to be useful, which was different from that of QVLM. Both numerical methods have given very close results with respect to both near- and far-field calculations of the induced drag. Moreover, they have agreed very well with the theoretical results presented in Ref. 4 rather than those based on Hauptman and Miloh's method.[9]

References

[1]Kalman, T. P., Giesing, J. P., and Rodden, W. P., "Spanwise Distribution of Induced Drag in Subsonic Flow by the Vortex Lattice Method," *Journal of Aircraft*, Vol. 7, No. 2, 1970, pp. 574–576.

[2]Lan, C. T., and Roskam, J., "Leading-Edge Force Features of the Aerodynamic Finite Element Method," *Journal of Aircraft*, Vol. 9, No. 12, 1972, pp. 864–867.

[3]Lan, C. E., "A Quasi-Vortex-Lattice Method in Thin Wing Theory," *Journal of Aircraft*, Vol. 11, No. 9, 1974, pp. 518–527.

[4]Ichikawa, M., "Analytical Expression of Induced Drag for a Finite Elliptic Wing," *Journal of Aircraft*, Vol. 33, No. 3, 1996, pp. 632–634.

[5]Kida, T., "A Theoretical Treatment of Lifting Surface Theory of an Elliptic Wing," *Zeitschrift fur Angewandte Mathematik und Mechanik*, Vol. 60, Dec. 1980, pp. 645–651.

[6]Ando, S., and Ichikawa, M., "A New Numerical Method of Subsonic Lifting Surfaces—BIS," *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 29, No. 84, 1986, pp. 101–118.

[7]Stark, V. J. E., "A Generalized Quadrature Formula for Cauchy Integrals," *AIAA Journal*, Vol. 9, No. 9, 1971, pp. 1854, 1855.

[8]DeJarnette, F. R., "Arrangement of Vortex Lattices on Subsonic Wings," *Vortex-Lattice Utilization*, NASA SP-405, N76-28180, Jan. 1976, pp. 301–323.

[9]Hauptman, A., and Miloh, T., "On the Exact Solution of the Linearized Lifting Surface Problem of an Elliptic Wing," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 39, Pt. 1, 1986, pp. 41–66.

Application of Wagner Functions in Symmetrical Airfoil Design

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Introduction

A FAIR amount of experience in airfoil design for various applications has been gained over the years. This is because the aircraft industry needs to develop products for new applications with an increased efficiency for achieving better economic results. Today the airfoil design procedure is being computerized. The specification of airfoil geometry in a suitable functional form is quite useful when implementing a computer code. Different methods for the design of airfoil shapes have already been developed or are currently under investigation.[1–3]

The literature survey indicates that there is a need to look for and try new functions to represent airfoil geometry. The functional relationship used should be numerically well conditioned as well as be suitable to represent a wide class of airfoil shapes. It should also have computational efficiency. In this study, the slope of the airfoil is represented by a Fourier series expansion of the Wagner functions.[4] It was found that the representation of airfoil contour in terms of Wagner functions seems to meet the previously mentioned requirements. Studies with Wagner functions showed that a large class of airfoils can be adequately described by a small number of terms. The unknown coefficients in the series representation are obtained by the least-square method. Standard NACA and NASA NFL(1)-00115 airfoil data[5–7] were used to test the suitability of Wagner function representation of airfoil shapes. It was observed that the results were quite satisfactory.

Analysis

Consider a symmetrical thin airfoil. The airfoil is assumed to be at zero angle attack. The distance along the chord line measured from the leading edge is denoted by x. Suppose that the slope of airfoil can be represented as follows:

$$f'(x) = -A_0 + \sum_{n=0}^{\infty} A_n H_n(\phi) \tag{1}$$

where ϕ is related to x by $1 - 2x = \cos\phi$ and $f'(x)$ is the slope of the airfoil. The Wagner functions are given by

$$H_n(\phi) = \frac{2}{\pi}\frac{\cos[(n+1)\phi] + \cos(n\phi)}{\sin\phi} \tag{2}$$

Equation (1) is integrated to obtain the airfoil contour and it can be represented by

$$f(\theta) = A_0\left[\left(\frac{\phi+\sin\phi}{\pi}\right) - \sin^2\frac{\phi}{2}\right] + \frac{1}{\pi}\sum_{n=1}^{\infty}\left\{\frac{\sin[(n+1)\phi]}{n+1} + \frac{\sin(n\phi)}{n}\right\} \tag{3}$$

Standard airfoil shapes are used to test the suitability of Eq. (3) in representing airfoil contours.

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Results and Discussion

Closed-form functional relationships are obtained to represent the various airfoil shapes by use of Wagner function series. The present method has the capability of generating a wide class of airfoil shapes with a small number of terms.

In the present approach, simple Wagner functions are used to characterize the airfoil shapes instead of using simple polynomials. The use of polynomials is undesirable since it requires too many parameters. The polynomials also require closure at the trailing edge of the contour. The Fourier series representation also has the same limitations as the polynomials. For these reasons, Wagner functions are used.

The basic idea here is to take a list of Wagner functions and try to find a linear combination of them that approximates the contour data as good as possible. A truncated form of Eq. (3) is used in fitting the NACA 0006, NACA 161015, and NASA NLF(1)-0115 airfoil contour data. A four-term Wagner function representation and actual data for various airfoil shapes are shown in Figs. 1 and 2 for the purpose of comparison. An examination of Figs. 1 and 2 shows that error in representing airfoil shapes by a four-term Wagner function representation is less than 1% in most instances. Highest error is seen at the leading and trailing edges. Wagner function series is also used to represent the upper and lower surface of NASA NLF(1)-0115 airfoils, and the results are shown in Figs. 3 and 4, respectively. A six-term Wagner function representation is used for the lower surface of an NLF(1)-0115 airfoil. A five-term Wagner function representation was found to be sufficient for the upper surface of the same airfoil. Again, error in representing the upper surface of the NASA NLF(1)-0115 by a five-term Wagner function expansion is around 1%. On the other hand, error in representing the lower surface of the NASA NLF(1)-0115 airfoil by a six-term Wagner function expansion is around 2%.

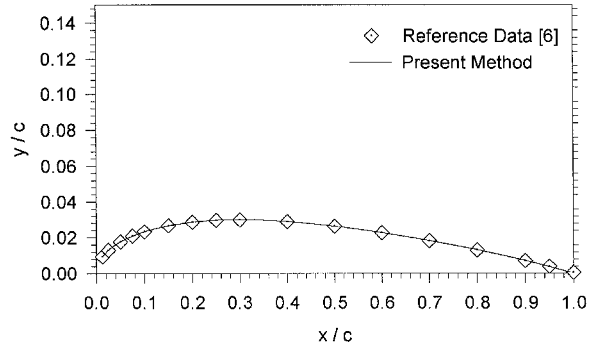

Fig. 1 NACA 0006 airfoil representation by Wagner function series.

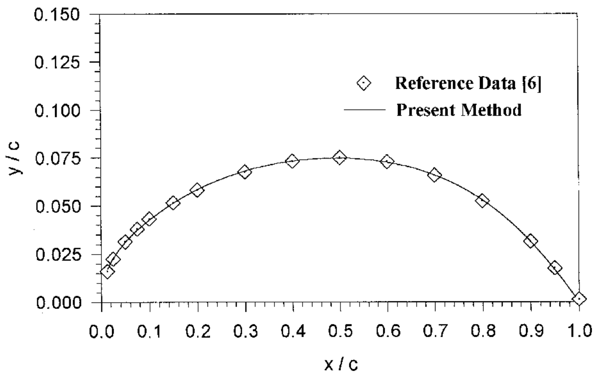

Fig. 2 NACA 16015 airfoil representation by Wagner function series.

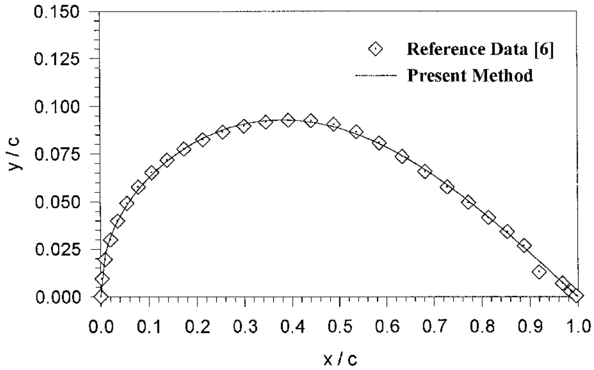

Fig. 3 Upper surface of NASA NLF(1)-0115 airfoil representation by Wagner function series.

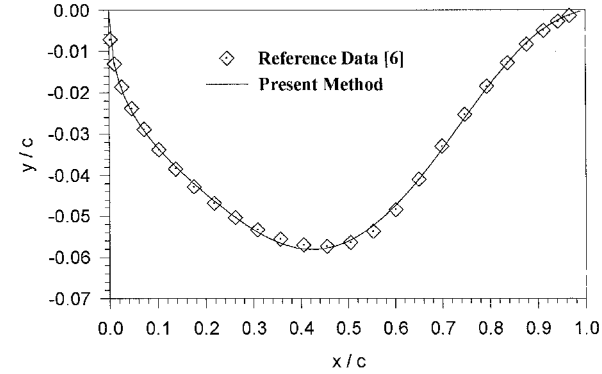

Fig. 4 Lower surface of NASA NLF(1)-0115 airfoil representation by Wagner function series.

Conclusions

A closed-form solution has been presented to represent airfoil shapes in terms of Wagner functions. It is clear from these results that a variety of airfoil shapes can be represented by a relatively small number of terms of a Wagner function series. Unknown coefficients of these terms are obtained by a least-square, curve-fitting procedure. Wagner function series representation was found to be quite useful when using thin airfoil theory. Wagner function series may also be used to represent the upper and lower surface of an airfoil. Nonsymmetrical airfoil shapes could also be represented by Wagner function series. The Wagner function representation of the airfoil allows fast calculations since all of the integrals are evaluated in closed form. It is also expected that the computational efficiency of design procedure can be improved by using a functional relationship representing airfoil shapes.

References

[1]Braembussche, R. V. D., "Special Course on Inverse Methods for Airfoil Design for Aeronautical and Turbomachinery Applications," AGARD-R-780, Nov. 1990.

[2]McGroskey, W. J., and Peckham, D. H., *Computational Methods for Aerodynamic Design (Inverse) and Optimization*, CP-463, AGARD, March 1990.

[3]Periaux, J., "Optimum Design Methods for Aerodynamics," AGARD-R-803, Nov. 1994.

[4]Ramamoorthy, P., Dwarakanath, G. S., and Narayana, C. L., "Wagner Functions," National Aeronautical Lab., TN 16, Bangalore 17, India, 1969.

[5]Clancy, L. J., *Aerodynamics*, Pitman Publishing Co., 1975.

[6]Abbot, I. H., and Von Doenhoff, A. E., *Theory of Wing Sections*, McGraw-Hill, New York, 1959.

[7]Seilig, M. S., Maughmert, M. D., and Somers, D. M., "Natural-Laminar-Flow Airflow for Generating-Aviation Applications," *Journal of Aircraft*, Vol. 23, No. 4, 1995, pp. 710-715.